Math 544, Spring 2002, Exam 1

PRINT Your Name:________________________________

There are 10 problems on 6 pages. Each problem is worth 5 points. SHOW your work. $\boxed{\textit{CIRCLE}}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

1. Find the general solution of the following system of linear equations:

$$\begin{array}{rrrcl} x_1 & +x_2 & +\ x_3 & = & 6 \\ & x_2 & +2x_3 & = & 5. \end{array}$$

2. Find the general solution of the following system of linear equations:

$$\begin{array}{rrrcl} x_1 & +\ x_2 & +\ x_3 & = & 6 \\ & x_2 & +2x_3 & = & 5 \\ x_1 & +2x_2 & +3x_3 & = & 10. \end{array}$$

3. Find the general solution of the following system of linear equations:

$$\begin{array}{rrcl} x_1 & +2x_2 & = & 3 \\ x_1 & +3x_2 & = & 2 \\ 3x_1 & +8x_2 & = & 7. \end{array}$$

4. Express $v = \begin{bmatrix} 3 \\ 2 \\ 7 \end{bmatrix}$ as a linear combination of $v_1 = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$ and $v_2 = \begin{bmatrix} 2 \\ 3 \\ 8 \end{bmatrix}$, if possible.

5. Define "linear combination". Use complete sentences.

6. Define "linearly independent". Use complete sentences.

7. Define "linear transformation". Use complete sentences.

8. Fill in the blank with an inequality involving m and p and then prove the result. Let $v_1, \dots, v_p$ be vectors in $\mathbb{R}^m$. If ________________________, then $v_1, \dots, v_p$ are linearly dependent.

9. True or False. (If true, explain why or give a proof. If false, give a counter example.) If v_1, v_2, v_3 are linearly independent vectors in $\mathbb{R}^4$ and $T\colon \mathbb{R}^4 \to \mathbb{R}^3$ is a linear transformation, then $T(v_1), T(v_2), T(v_3)$ are linearly independent vectors in $\mathbb{R}^3$.

10. True or False. (If true, explain why or give a proof. If false, give a counter example.) If v_1, v_2, v_3 are linearly independent vectors in $\mathbb{R}^4$ and v_4 is a vector in $\mathbb{R}^4$ which is not a linear combination of v_1, v_2, and v_3, then v_1, v_2, v_3, v_4 are linearly independent vectors in $\mathbb{R}^4$.