PRINT Your Name:________________________________

Quiz for May 30, 2012

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.** SHOW your work. Express your work in a neat and coherent manner. $\boxed{\text{BOX}}$ your answer.
(Recall that the matrix M is symmetric if $M^{\mathrm{T}} = M$.)

Let A and B be $n \times n$ symmetric matrices. **State** a necessary and sufficient condition for the matrix AB to be symmetric. **Prove** both directions of your assertion.

(You are supposed to state a true fact that looks like AB is symmetric if and only if XXX. Then you are supposed to prove that if AB is symmetric, then XXX happens. Then you are supposed to prove that if XXX happens, then AB is symmetric. Of course, XXX is more interesting than merely, "AB is symmetric".)

ANSWER: The matrix AB is symmetric if and only if $AB = BA$.

($\Rightarrow$) Assume that AB is symmetric. We must show that $AB = BA$. We know that

$$AB = (AB)^{\mathrm{T}} = B^{\mathrm{T}} A^{\mathrm{T}} = BA.$$

The first equality holds because AB is symmetric. The second equality was estabished in class on Tuesday. The third equality holds because A and B are both symmetric.

($\Leftarrow$) Assume that $AB = BA$. We must show that AB is symmetric. We know that

$$(AB)^{\mathrm{T}} = (BA)^{\mathrm{T}} = A^{\mathrm{T}} B^{\mathrm{T}} = AB.$$

The first equality holds because $AB = BA$ is symmetric. The second equality was estabished in class on Tuesday. The third equality holds because A and B are both symmetric.