PRINT Your Name:________________________________

Quiz for May 30, 2012

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.** Write in complete sentences. Express your work in a neat and coherent manner.

Let A and B be $n \times n$ matrices show that AB is non-singular. **Prove** that A is non-singular.

ANSWER: We first show that B is non-singular. Suppose that v is a vector with $Bv = 0$. Multiply both sides of the equation on the left by A to get $A(Bv) = B0$. It follows that $(AB)v = 0$. The matrix AB is non-singular; hence, $v = 0$.

The matrix B is non-singular; hence, according to the non-singular matrix theorem, B has an inverse. Let C be B's inverse.

Now we show that A is non-singular. Suppose v is a vector with $Av = 0$. We know that $BC = I$. We have $0 = Av = AB(Cv)$. The matric AB is non-singular by hypothesis; hence, the vector Cv is equal to zero. Multiply $Cv = 0$ by B to conclude $BCv = 0$. But, $BC = I$ and therefore, $v = 0$.