PRINT Your Name:__________________________________

Quiz for June 21, 2012

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.** Write in complete sentences. Express your work in a neat and coherent manner.

Recall that $\mathcal{P}_4$ is the vector space of polynomials of degree less than or equal to 4. Let W be the subspace of $\mathcal{P}_4$ which is defined as follows: the polynomial $p(x)$ is in W if and only if $p(1)+p(-1)=0$ and $p(2)+p(-2)=0$. Find the dimension of W. Explain.

ANSWER: Consider the linear transformation $T\colon \mathcal{P}_4 \to \mathbb{R}^2$, which is given by $T(p(x)) = \begin{bmatrix} p(1)+p(-1) \\ p(2)+p(-2) \end{bmatrix}$. The vector space W is the null space of T. So the dimension of W is equal to the dimension of $\mathcal{P}_4$ minus the dimension of the image of T. We know that $\dim \mathcal{P}_4 = 5$, since $1, x, x^2, x^3, x^4$ is a basis for $\mathcal{P}_4$. The image of T is all of $\mathbb{R}^2$ because the image of T is a subspace of $\mathbb{R}^2$ which contains $T(1) = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$ and $T(x^2) = \begin{bmatrix} 2 \\ 8 \end{bmatrix}$. The vectors $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 8 \end{bmatrix}$ span $\mathbb{R}^2$. We conclude that

$$\dim W = 5 - 2 = \boxed{3}.$$

(There are many other ways to reach this answer. The most straightforward thing to do is to calculate a basis for W.)