PRINT Your Name:_____

<div align="center">**Quiz for June 11, 2012**</div>

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.** Write in complete sentences. Express your work in a neat and coherent manner.

Let W be the vector space which consists of all vectors $\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$ in $\mathbb{R}^4$ with

$$\begin{aligned} x_1 + x_2 - x_3 \quad\;\;\; &= 0 \\ x_2 \quad\;\; - x_4 &= 0 \end{aligned}$$

Find a basis for W.

ANSWER: The vector space W is the null space of $\begin{bmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}$. We apply Gauss Jordan Elimination. Replace Row 1 with Row 1 minus Row 2 to obtain $\begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix}$. This matrix is in Reduced Row-Echelon Form. We read that W is the set of all

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

with

$$\begin{aligned} x_1 &= x_3 - x_4 \\ x_2 &= \quad\;\; x_4 \\ x_3 &= x_3 \\ x_4 &= \quad\;\; x_4 \end{aligned}$$

In other words,

$$\boxed{w_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, w_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}}$$

is a basis for W.