PRINT Your Name:___________________________________

Quiz for June 11, 2012

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil.** Write in complete sentences. Express your work in a neat and coherent manner.

Let U and V be vector spaces which both are subsets of a vector space W. Let

$$S = \{w \in W \mid w = u + v \quad \text{for some } u \in U \text{ and } v \in V\}.$$

Prove that S is a vector space.

ANSWER:

The zero vector W is in S. The zero vector of W is in U because U is a vector space. The zero vector of W is in V because V is a vector space. Thus, $0 + 0 = 0$ is in S. (The first zero on the left side of the equation is 0 from U; the second zero is the 0 from V.)

S is closed under addition: Take s_1 and s_2 from S. It follows that $s_1 = u_1 + v_1$ and $s_2 = u_2 + v_2$ for some u_i in U and $v_i \in V$. Observe that

$$s_1 + s_2 = (u_1 + u_2) + (v_1 + v_2)$$

is in S since $u_1 + u_2$ is in U (because U is a vector space) and $v_1 + v_2$ is in V (because V is a vector space).

S is closed under addition: Take s_1 from S and c from $\mathbb{R}$. It follows that $s_1 = u_1 + v_1$ for some u_1 in U and $v_1 \in V$. Observe that

$$cs_1 = (cu_1) + (cv_1)$$

is in S since cu_1 is in U (because U is a vector space) and cv_1 is in V (because V is a vector space).