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Quiz for September 24, 2009

Suppose A and B are $n \times n$ matrices with $AB = I$. Does BA have to equal I? If yes, give a proof. If no, give an example.

ANSWER: YES. The hypothesis $AB = I$ guarantees that B is non-singular because if x is a vector with $Bx = 0$, then we multiply both sides by A to learn $x = ABx = A0 = 0$. The only vector that B sends to zero is $x = 0$. The non-singular matrix theorem (see, for example, problem 3 on Exam 1) guarantees that B is invertible. So there is a matrix B^{-1} with $BB^{-1} = I$ and $B^{-1}B = I$. Multiply both sides of $AB = I$ on the right with B^{-1} to get

$$A = ABB^{-1} = IB^{-1} = B^{-1}.$$

We conclude that

$$BA = BB^{-1} = I.$$