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**Quiz for June 22, 2006**

Let  $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$  be a linear transformation. Suppose that

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}.$$

Find  $T\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}\right)$ . Explain.

**ANSWER:** We see that

$$\begin{bmatrix} 2 \\ -1 \end{bmatrix} = 2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} - 1 \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

The hypothesis that  $T$  is a linear transformation says that

$$\begin{aligned} T\left(\begin{bmatrix} 2 \\ -1 \end{bmatrix}\right) &= T\left(2 \begin{bmatrix} 1 \\ 0 \end{bmatrix} - 1 \begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = 2T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) - 1T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) \\ &= 2 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} - 1 \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} = \boxed{\begin{bmatrix} 0 \\ -1 \\ -2 \end{bmatrix}}. \end{aligned}$$