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Quiz for June 14, 2006

Let A be an $m \times m$ non-singular matrix and let B be an $m \times n$ matrix. Prove that the null space of AB is equal to the null space of B.

ANSWER: First we prove that the null space of B is contained in the null space of AB. (This part of the argument does not require that A be non-singular.) We are given a vector v in $\mathbb{R}^n$ with $Bv = 0$. Multiply both sides of the equation by A to get $ABv = A0 = 0$ to conclude that v is in the null space of AB.

Now we prove that the null space of AB is contained in the null space of B. We start with a vector v in $\mathbb{R}^n$ with $ABv = 0$. The matrix A is non-singular, so the only way for $A(Bv)$ to equal zero is if Bv is already zero. Thus, v is in the null space of B.