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Quiz for October 29, 2009

Let A be an $m \times m$ nonsingular matrix and let B be an $m \times n$ matrix. Prove that AB and B have the same rank.

ANSWER: We first prove that AB and B have the same null space.

The null space of B is contained in the null space of AB: Take v in the null space of B. Thus, $Bv = 0$. Multiply by A to see that $ABv = 0$. Thus, v is in the null space of AB.

The null space of AB is contained in the null space of B: Take v in the null space of AB. Thus, $ABv = 0$. Multiply by A^{-1} to see that $A^{-1}ABv = 0$. Thus, $Bv = 0$ and v is in the null space of B.

Now we finish the proof: Use the rank-nullity Theorem. The rank of AB is equal to the number of columns of AB minus the nullity of AB. The number of columns of AB is the same as the number of columns of B, and the nullity of AB is the same as the nullity of B. So the rank of AB is equal to the number of columns of B minus the nullity of B, and this, by the rank-nullity Theorem, is the rank of B.