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Quiz for October 15, 2009

Let W be the subspace of $\mathbb{R}^4$ defined by

$$W = \{x \mid a^{\mathrm{T}}x = 0 \quad \text{and} \quad b^{\mathrm{T}}x = 0 \quad \text{and} \quad c^{\mathrm{T}}x = 0\}.$$

Calculate the dimension of W for

$$a = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \quad \text{and} \quad c = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix}.$$

ANSWER: We see that W is the null space of

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix}.$$

We have an algorithm for finding the basis of a null space. Subtract row 1 from row 2:

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix}$$

Subtract row 2 from row 3.
Add row 2 to row 1.

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The null space of A is the set of vectors

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

with

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

for some numbers x_3 and x_4. Thus,

$$\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

is a basis for W and $\boxed{\dim W = 2}$.