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Quiz for October 1, 2009

Let W be a subspace of $\mathbb{R}^n$ and let A be any $m \times n$ matrix. Let V be the subset of $\mathbb{R}^m$ defined by

$$V = \{y \in \mathbb{R}^m \mid y = Ax \text{ for some } x \text{ in } W\}.$$

Prove that V is a subspace of $\mathbb{R}^m$.

ANSWER:

• Zero is in V. The set W is a vector space; so $0 \in W$ and $A0 = 0$ is in V.

• The set V is closed under addition. Take v_1 and v_2 in V. The definition of V says that there are w_1 and w_2 in W with $v_i = Aw_i$ for both i. The set W is a vector space; so W is closed under addition. Thus, $w_1 + w_2 \in W$ and $A(w_1 + w_2)$ is in V. On the other hand $A(w_1 + w_2) = Aw_1 + Aw_2 = v_1 + v_2$. We have shown that $v_1 + v_2$ is in V.

• The set V is closed under scalar multiplication. Take v in V and c in $\mathbb{R}$. The definition of V says that there is a vector w in W with $Aw = v$. The set W is a vector space; so W is closed under scalar multiplication and therefore, cw is in W. It follows that $A(cw)$ is in V. On the other hand, $A(cw) = cAw = cv$. We have shown that cv is in V.