Quiz 7 Math 544, Monday, November 2, 2020

Let

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 & 1 & 3 \\ 2 & 4 & 6 & 2 & 1 & 5 \\ 2 & 4 & 6 & 1 & 2 & 5 \\ 2 & 4 & 6 & 1 & 1 & 4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 2 & 3 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

I applied Elementary Row Operations to obtain the matrix B from the matrix A.
Please do NOT verify this assertion.

(a) Find a basis for the null space of A.

The vectors

$$\boxed{w_1 = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad w_2 = \begin{bmatrix} -3 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad w_3 = \begin{bmatrix} -1 \\ 0 \\ 0 \\ -1 \\ -1 \\ 1 \end{bmatrix}}$$

are a basis for the null space of A.

(b) Find a basis for the column space of A.

The vectors

$$\boxed{A_{*,1} = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 2 \end{bmatrix}, \quad A_{*,4} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \quad A_{*,5} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}}$$

are a basis for the column space of A. Notice that I am writing $A_{*,j}$ for column j of the matrix A.

(c) Find a basis for the row space of A.

The vectors

$$\boxed{\begin{aligned} z_1 &= \begin{bmatrix} 1 & 2 & 3 & 0 & 0 & 1 \end{bmatrix} \\ z_2 &= \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix} \\ z_3 &= \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{aligned}}$$

are a basis for the row space of A.

(d) Express each column of A in terms of your answer to (b).

We see that

$$\boxed{A_{*,2} = 2A_{*,1}, \quad A_{*,3} = 3A_{*,1}, \quad A_{*,6} = A_{*,1} + A_{*,4} + A_{*,5}.}$$

(e) Express each row of A in terms of your answer to (c).

I write $A_{i,*}$ for row i of A. We see that

$$\boxed{\begin{aligned} A_{1,*} &= z_1 + z_2 + z_3, \\ A_{2,*} &= 2z_1 + 2z_2 + z_3, \\ A_{3,*} &= 2z_1 + z_2 + 2z_3, \\ A_{4,*} &= 2z_1 + z_2 + z_3. \end{aligned}}$$