Please PRINT your name ______________________________________

No calculators, cell phones, computers, notes, etc.

Circle your answer. $\boxed{\text{Make your work correct, complete and coherent.}}$

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you. I will keep your quiz.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 6, March 28, 2022

The *trace* of the square matrix A is the sum of the numbers on its main diagonal. Let V be the set of all 3×3 matrices with trace 0. The set V is a vector space. You do NOT have to prove this. Give a basis for V. Prove that your proposed basis really is a basis.

Answer: The matrices

$$M_1=\begin{bmatrix}1&0&0\\0&-1&0\\0&0&0\end{bmatrix},\quad M_2=\begin{bmatrix}1&0&0\\0&0&0\\0&0&-1\end{bmatrix},\quad M_3=\begin{bmatrix}0&1&0\\0&0&0\\0&0&0\end{bmatrix},$$

$$M_4=\begin{bmatrix}0&0&1\\0&0&0\\0&0&0\end{bmatrix},\quad M_5\begin{bmatrix}0&0&0\\1&0&0\\0&0&0\end{bmatrix},\quad M_6=\begin{bmatrix}0&0&0\\0&0&1\\0&0&0\end{bmatrix},$$

$$M_7=\begin{bmatrix}0&0&0\\0&0&0\\1&0&0\end{bmatrix},\quad M_8=\begin{bmatrix}0&0&0\\0&0&0\\0&1&0\end{bmatrix}$$

are are a basis for V.

The proposed basis in linearly independent. If $c_1,\dots,c_8$ are numbers with $\sum_{i=1}^{8} c_iM_i$ equal to the zero matrix, then

$$\begin{bmatrix}c_1+c_2&c_3&c_4\\c_5&-c_1&c_6\\c_7&c_8&-c_2\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix};$$

hence all eight c's are zero.

The proposed basis spans V. A typical element of V looks like

$$A=\begin{bmatrix}a_{11}&a_{12}&a_{13}\\a_{21}&a_{22}&a_{23}\\a_{31}&a_{32}&a_{33}\end{bmatrix},$$

where $a_{11}+a_{22}+a_{33}=0$. Observe that

$$A=-a_{22}M_1-a_{33}M_2+a_{12}M_3+a_{13}M_4+a_{21}M_5+a_{23}M_6+a_{31}M_7+a_{32}M_8.$$