Quiz 5 Math 544, Monday, October 12, 2020

Let A be an $m \times r$ matrix and B be an $r \times n$ matrix.

(a) How is the null space of B related to the null space of AB? Prove your answer.

(b) How is the column space of A related to the column space of AB? Prove your answer.

Answer:

Observation. The null space of B is contained in the null space of AB.

Proof. If v is in the null space of B, then $Bv = 0$. Apply A to both sides to see that $ABv = A0 = 0$. Conclude that v is in the null space of AB. □

Observation. The column space of AB is contained in the column space of A.

Proof. If w is in the column space of AB, then there is a vector $v \in \mathbb{R}^n$ with $(AB)v = w$. Hence $A(Bv) = w$ and w is in the column space of A. □