Please PRINT your name ______________________________________

No calculators, cell phones, computers, notes, etc.

Circle your answer. $\boxed{\text{Make your work correct, complete and coherent.}}$

Please take a picture of your quiz (for your records) just before you turn the quiz in. I will e-mail your grade and my comments to you. I will keep your quiz.

The quiz is worth 5 points. The solutions will be posted on my website later today.

Quiz 3, February 16, 2022

Let v_1, v_2, and v_3 be vectors in $\mathbb{R}^n$ and M be a nonsingular $n \times n$ matrix. Suppose the vectors v_1, v_2, v_3 are linearly independent. Do the vectors Mv_1, Mv_2, Mv_3 have to be linearly independent? If yes, prove your answer. If no, give a counterexample.

Answer: The vectors Mv_1, Mv_2, Mv_3 are linearly independent.

Proof. Suppose c_1, c_2, c_3 are numbers with

$$c_1Mv_1 + c_2Mv_2 + c_3Mv_3 = 0.$$

Use the property of scalars and the fact that matrix multiplication distributes over addition to see that

$$M(c_1v_1 + c_2v_2 + c_3v_3) = 0.$$

The matrix M is nonsingular; hence, the only vector w with $Mw = 0$ is $w = 0$. Thus, $c_1v_1 + c_2v_2 + c_3v_3 = 0$. On the other hand, the vectors v_1, v_2, v_3 are linearly independent. It follows that c_1, c_2, c_3 must all be zero. We have proven tha t Mv_1, Mv_2, Mv_3 are linearly independent. $\square$