

Quiz 2 Math 544, September 14, 2020

Let $v_1, v_2,$ and v_3 be vectors in \mathbb{R}^n and M be an $n \times n$ matrix. Suppose the vectors Mv_1, Mv_2, Mv_3 are linearly independent. Do the vectors v_1, v_2, v_3 have to be linearly independent? If yes, prove your answer. If no, give a counterexample.

ANSWER: YES! Let $c_1, c_2,$ and c_3 be numbers with $c_1v_1 + c_2v_2 + c_3v_3 = 0$. Multiply by M and distribute to see that $c_1Mv_1 + c_2Mv_2 + c_3Mv_3 = 0$. The vectors Mv_1, Mv_2, Mv_3 are linearly independent; hence, $c_1 = c_2 = c_3 = 0$.