Math 544, Exam 4, Fall 2006

Write your answers as legibly as you can on the blank sheets of paper provided.

Please leave room in the upper left corner for the staple.

Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you.

There are 10 problems. Each problem is worth 5 points. The exam is worth a total of 50 points.

SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators or Cell phones.**

I will post the solutions on my website sometime this afternoon.

If I know your e-mail address, I will e-mail your grade to you as soon as I have graded the exam. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

1. Define "linear transformation". Use complete sentences. Include everything that is necessary, but nothing more.

2. Define "dimension". Use complete sentences. Include everything that is necessary, but nothing more.

3. Find all eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} -1 & -6 \\ 1 & 4 \end{bmatrix}$. **CHECK** your answer

4. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be reflection across the line $y = \sqrt{3}x$. Find the matrix M with $T(v) = Mv$ for all $v \in \mathbb{R}^2$.

5. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation. Suppose $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ and $T\left(\begin{bmatrix} 2 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$. Find the matrix M with $T(v) = Mv$ for all $v \in \mathbb{R}^2$.

6. Give an example of a NON-ZERO 2×2 matrix A whose only eigenvalue is zero.

7. Solve $Ax = b$ for

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}.$$

   You might want to notice that the columns of A form an orthogonal set. **CHECK** your answer.

8. Suppose that the non-zero vectors v_1, v_2, v_3 form an orthogonal set. Prove that v_1, v_2, v_3 are linearly independent. Give **a complete proof**.

9. Suppose $V_1 \subseteq V_2 \subseteq V_3$ are vector spaces and v_1, v_2, v_3, v_4 are vectors in V_3 which form a basis for V_3. Suppose further, that v_1, v_2, v_3 are in V_2 and $v_4 \notin V_2$. Suppose v_1, v_2 are in V_1 and $v_3 \notin V_1$. Do you have enough information to know the dimension of V_1. **Explain very thoroghly.**

10. Find an orthogonal basis for the null space of $A = [1 \quad 2 \quad 3 \quad 5]$. **CHECK your answer.**