Math 544, Exam 3, Fall 2006

Write your answers as legibly as you can on the blank sheets of paper provided.

Please leave room in the upper left corner for the staple.

Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you.

The exam is worth a total of 50 points.

No Calculators or Cell phones.

I will post the solutions on my website sometime this afternoon.

If I know your e-mail address, I will e-mail your grade to you as soon as I have graded the exam. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

1. (5 points) Define "closed under addition". Use complete sentences. Include everything that is necessary, but nothing more.

2. (5 points) Define "basis". Use complete sentences. Include everything that is necessary, but nothing more.

3. (5 points) Let A be an $n \times n$ matrix and let $W = \{v \in \mathbb{R}^n | \, Av = 2v\}$. Is W a subspace of $\mathbb{R}^n$? If yes, then give a complete, correct, proof. If no, then give an explicit example that shows that W is not a subspace of of $\mathbb{R}^n$.

4. (5 points) Let U and V be subspaces of $\mathbb{R}^n$. Is the union $U \cup V$ a subspace of $\mathbb{R}^n$? If yes, then give a complete, correct, proof. If no, then give an explicit example that shows that $U \cup V$ is not a subspace of $\mathbb{R}^n$.

5. (10 points) State the four theorems about dimension.

6. (10 points) Let V and W be subspaces of $\mathbb{R}^n$ with $V \subseteq W$.
   (a) Does the dimension of V have to be $\leq$ the dimension of W ? If yes, then give a complete, correct, proof. If no, then give an explicit example.
   (b) Suppose $\dim V = \dim W$. Does V have to equal W ? If yes, then give a complete, correct, proof. If no, then give an explicit example.

7. (10 points) Let A and B be $n \times n$ matrices, with A non-singular. Answer each question. If the answer is yes, then give a complete, correct, proof. If the answer is no, then give an example.
   (a) Does the null space of B have to be equal the null space of AB?
   (b) Does the dimension of the null space of B have to equal the dimension of the null space of AB?
   (c) Does the column space of B have to equal the column space of AB?
   (d) Does the dimension of the column space of B have to equal the dimension of the column space of AB?