Math 544, Exam 2, Fall 2006

Write your answers as legibly as you can on the blank sheets of paper provided.

Please leave room in the upper left corner for the staple.

Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you.

The exam is worth a total of 50 points. There are 9 problems. Problem 9 is worth 10 points. Every other problem is worth 5 points.

No Calculators or Cell phones.

I will post the solutions on my website sometime this afternoon.

If I know your e-mail address, I will e-mail your grade to you as soon as I have graded the exam. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

1. Define "null space". Use complete sentences. Include everything that is necessary, but nothing more.

2. Define "linearly independent". Use complete sentences. Include everything that is necessary, but nothing more.

3. Define "subspace of $\mathbb{R}^n$". Use complete sentences. Include **everything** that is necessary, but nothing more.

4. Let $W = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2 \,\middle|\, |x_1| = |x_2| \right\}$. Is W a subspace of $\mathbb{R}^2$? If yes, then give a complete, correct, proof. If no, then give an explicit example that shows that W is not a subspace of of $\mathbb{R}^2$.

5. Let $W = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \in \mathbb{R}^4 \,\middle|\, \begin{matrix} x_1 + 2x_2 = 3x_3 \\ 2x_1 - 3x_2 = 4x_4 \end{matrix} \right\}$. Is W a subspace of $\mathbb{R}^4$? If yes, then give a complete, correct, proof. If no, then give an explicit example that shows that W is not a subspace of of $\mathbb{R}^4$.

6. Let v_1, v_2, v_3, v_4 be vectors in $\mathbb{R}^5$, with v_1, v_2, v_3 linearly independent and $v_4 = 2v_1 + 3v_2 + 4v_3$. Do the vectors v_2, v_3, v_4 have to be linearly independent? If yes, then give a complete, correct, proof. If no, then give an example.

7. Let v_1, v_2, v_3, v_4 be vectors in $\mathbb{R}^5$, with v_1, v_2, v_3 linearly independent and v_4 equal to a linear combination of v_1, v_2, and v_3. Do the vectors v_2, v_3, v_4 have to be linearly independent? If yes, then give a complete, correct, proof. If no, then give an example.

8. How many solutions does a system of 4 linear equations in 3 unknowns have? Justify your answer very thoroughly.

9. Let A and B be $n \times n$ matrices. Answer each question. If the answer is yes, then give a complete, correct, proof. If the answer is no, then give an example.
   (a) Does the null space of A have to be a subset of the null space of AB?
   (b) Does the null space of B have to be a subset of the null space of AB?
   (c) Does the null space of AB have to be a subset of the null space of A?
   (d) Does the null space of AB have to be a subset of the null space of B?