Math 544,   Exam 1,   Fall 2006

Write your answers as legibly as you can on the blank sheets of paper provided.

Please leave room in the upper left corner for the staple.

Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you.

The exam is worth a total of 50 points. There are 10 problems. Each problem is worth 5 points.

SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators or Cell phones.**

I will post the solutions on my website sometime Wednesday afternoon.

I will grade the exam Wednesday afternoon. If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

1. Find the GENERAL solution of the system of linear equations $Ax = b$. Also, list three SPECIFIC solutions, if possible. CHECK that the specific solutions satisfy the equations.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 1 & 6 \\ 1 & 2 & 3 & 4 & 2 & 12 \\ 2 & 4 & 6 & 8 & 3 & 18 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 5 \\ 8 \end{bmatrix}.$$

2. Consider the system of linear equations.

$$\begin{aligned} x_1 + (a-1)x_2 &= 4 \\ ax_1 + \quad\ 6x_2 &= 12. \end{aligned}$$

 (a) Which values for a cause the system to have no solution?
 (b) Which values for a cause the system to have exactly one solution?
 (c) Which values for a cause the system to have an infinite number of solutions?

Explain thoroughly.

3. Are the vectors

$$v_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

   linearly independent? **Explain thoroughly.**

4. Suppose v_1, v_2 and v_3 are vectors in $\mathbb{R}^3$ with v_1, v_2 linearly independent, v_1, v_3 linearly independent, and v_2, v_3 linearly independent. Do the vectors v_1, v_2, v_3 have to be linearly independent? If yes, give a proof. If no, give an example.

5. How many solutions does a homogeneous system of 3 linear equations in 4 unknowns have? Justify your answer very thoroughly.

6. How many solutions does a homogeneous system of 4 linear equations in 3 unknowns have? Justify your answer very thoroughly.

7. Recall that the matrix A is *symmetric* if $A^{\mathrm{T}} = A$. Let A and B be 2×2 symmetric matrices. Give an example to show that AB does not have to be a symmetric matrix.

8. Give a condition (*) so that if A and B are 2×2 symmetric matrices which satisfy (*), then AB also is a symmetric matrix.

9. List four different 2×2 matrices X which satisfy $X^2 - 2X = 0$.

10. Find a matrix B with $AB = C$ for $A = \begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix}$ and $C = \begin{bmatrix} 2 & 6 \\ 3 & 6 \end{bmatrix}$.