Math 544, Exam 4, Fall 2005

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you. There are 10 problems. Each problem is worth 5 points. The exam worth 50 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

I will post the solutions on my website shortly after the exam is finished.

Recall that the matrix A is *symmetric* if $A^{\mathrm{T}} = A$ and the matrix A is *skew-symmetric* if $A^{\mathrm{T}} = -A$.

1. Let A be a 2×2 symmetric matrix with real number entries. Does A HAVE to have real eigenvalues? If yes, PROVE it. If no, then give a counter example.

2. Let A be a 2×2 skew-symmetric matrix with real number entries. Does A HAVE to have real eigenvalues? If yes, PROVE it. If no, then give a counter example.

3. Let A be a 3×3 skew-symmetric matrix with real number entries. Does A have to be singular? If yes, PROVE it. If no, then give a counter example.

4. Let A be a 2×2 skew-symmetric matrix with real number entries. Does A have to be singular? If yes, PROVE it. If no, then give a counter example.

5. Find the inverse of
$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & -1 & 1 \\ 1 & -1 & 1 & 0 \\ 1 & 0 & -1 & -1 \end{bmatrix}.$$
It might be worth your while to notice that the columns of A form an orthogonal set.

6. Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation with $T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ and $T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right) = \begin{bmatrix} 6 \\ 7 \end{bmatrix}$. Find a matrix M with $T(v) = Mv$ for all vectors v in $\mathbb{R}^2$.

7. Let V be the set of 2×2 singular matrices. Is V a vector space? Explain **thoroughly**.

8. Let V be the set of 2×2 matrices with trace 0. Is V a vector space? Explain **thoroughly**. Recall that the *trace* of a square matrix is the sum of the elements on its main diagonal.

9. Find a matrix B with $B^2 = A$ for $A = \begin{bmatrix} 34 & 75 \\ -10 & -21 \end{bmatrix}$. **Check your answer.**

10. Let A be a square matrix, v_1 and v_2 be non-zero vectors with $Av_1 = \lambda_1 v_1$ and $Av_2 = \lambda_2 v_2$, where λ_1 and λ_2 are real numbers with $\lambda_1 \neq \lambda_2$. Prove that $\{v_1, v_2\}$ is a linearly independent set of vectors.