Math 544, Exam 3, Fall 2005

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you. There are 10 problems. The exam worth 50 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

I will post the solutions on my website shortly after the exam is finished.

1. (8 points) Let

$$A = \begin{bmatrix} 2 & 6 & 2 & 8 & 2 \\ 2 & 6 & 3 & 11 & 2 \\ 4 & 12 & 5 & 19 & 5 \\ 2 & 6 & 2 & 8 & 2 \end{bmatrix}.$$

   (a) Find a basis for the null space of A.
   (b) Find a basis for the column space of A.
   (c) Find a basis for the row space of A.
   (d) Write each column of A as a linear combination of your answer to (b).
   (e) Write each row of A as a linear combination of your answer to (c).

2. (6 points) Find an orthogonal basis for the vector space spanned by

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

3. (4 points) Express $v = \begin{bmatrix} 3 \\ 2 \\ 0 \\ -1 \end{bmatrix}$ as a linear combination of

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \quad \text{and } v_3 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}.$$

   You are encouraged to notice that v_1, v_2, v_3 is an orthogonal set.

4. (8 points) Let A be a non-singular $n \times n$ matrix and B be an $n \times n$ matrix? Answer each question. If the answer is "yes", prove the statement. If the answer is "no", give an example.
(a) Does the column space of AB have to equal the column space of B?
(b) Does the null space of AB have to equal the null space of B?
(a) Does the rank of AB have to equal the rank of B?

5. (4 points) Let

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \quad v_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad v_4 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

Let V be a subspace of $\mathbb{R}^4$. Suppose that $v_1 \in V$, $v_2 \in V$, $v_3 \notin V$, and $v_4 \notin V$. Do you have enough information to determine the dimension of V? Explain very thoroughly.

6. (4 points) State any one of the four dimension Theorems.

7. (4 points) Define "basis". Use complete sentences. Include everything that is necessary, but nothing more.

8. (4 points) Define "dimension". Use complete sentences. Include everything that is necessary, but nothing more.

9. (4 points) Let $V = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 \;\middle|\; \begin{array}{l} x_1 + 3x_2 + 4x_3 = 1 \\ 2x_1 + 9x_2 + 5x_3 = 0 \\ 5x_1 + 14x_2 + 41x_3 = 0 \\ -x_1 + 32x_2 + 12x_3 = 0 \end{array} \right\}$. Is V a vector space? Explain **thoroughly**.

10. (4 points) Let $V = \left\{ \begin{bmatrix} x_1 + 3x_2 + 4x_3 \\ 2x_1 + 9x_2 + 5x_3 \\ 5x_1 + 14x_2 + 41x_3 \\ -x_1 + 32x_2 + 12x_3 \end{bmatrix} \in \mathbb{R}^4 \;\middle|\; x_1, x_2, x_3 \in \mathbb{R} \right\}$. Is V a vector space? Explain **thoroughly**.