Math 544, Exam 2, Fall 2005

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you. There are 10 problems. Each problem is worth 5 points. SHOW your work. $\boxed{\mathit{CIRCLE}}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

If I know your e-mail address, I will e-mail your grade to you. If I don't already know your e-mail address and you want me to know it, then **send me an e-mail**.

I will post the solutions on my website shortly after the exam is finished.

1. Find the GENERAL solution of the system of linear equations $Ax = b$. Also, list three SPECIFIC solutions, if possible. CHECK that the specific solutions satisfy the equations.

$$A = \begin{bmatrix} 1 & 2 & 2 & 10 \\ 1 & 2 & 3 & 13 \\ 2 & 4 & 5 & 23 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \quad b = \begin{bmatrix} -1 \\ -2 \\ -3 \end{bmatrix}.$$

2. Let U and V be subspaces of $\mathbb{R}^n$. Does the **union** $U \cup V$ have to be a subspace of $\mathbb{R}^n$? If yes, prove your answer. If no, give a counterexample.

3. Let $V = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 \;\middle|\; \begin{matrix} x_1 + 3x_2 + 4x_3 = 0 \\ 2x_1 + 9x_2 + 5x_3 = 0 \\ 5x_1 + 14x_2 + 41x_3 = 0 \\ -x_1 + 32x_2 + 12x_3 = 0 \end{matrix} \right\}$. Is V a vector space? Explain **thoroughly**.

4. Let $V = \left\{ \begin{bmatrix} x_1 + 3x_2 + 4x_3 \\ 2x_1 + 9x_2 + 5x_3 \\ 5x_1 + 14x_2 + 41x_3 \\ -x_1 + 32x_2 + 12x_3 \end{bmatrix} \in \mathbb{R}^4 \;\middle|\; x_1, x_2, x_3 \in \mathbb{R} \right\}$. Is V a vector space? Explain **thoroughly**.

5. Let $V = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{R}^2 \;\middle|\; x_1 x_2 = 0 \right\}$. Is V a vector space? Explain **thoroughly**.

6. Define "null space". Use complete sentences. Include everything that is necessary, but nothing more.

7.
   (a) Define "non-singular". Use complete sentences. Include everything that is necessary, but nothing more.
   (b) Let A be an $n \times n$ matrix. List three statements that are equivalent to the statement "A is non-singular".

8. Let A and B be 2×2 matrices with A not equal to the zero matrix and $A^2 = AB$. Does A have to equal B? If yes, prove your answer. If no, give a counterexample.

9. Let A and B be $n \times n$ matrices. At least one of the following statements is always true. Pick a true statement and prove it.
   (a) The column space of A is a subset of the column space of AB.
   (b) The column space of B is a subset of the column space of AB.
   (c) The column space of AB is a subset of the column space of A.
   (d) The column space of AB is a subset of the column space of B.

10. Let A and B be $n \times n$ matrices. At least one of the following statements is always true. Pick a true statement and prove it.
   (a) The null space of A is a subset of the null space of AB.
   (b) The null space of B is a subset of the null space of AB.
   (c) The null space of AB is a subset of the null space of A.
   (d) The null space of AB is a subset of the null space of B.