Math 544, Exam 1, Fall 2005

Write your answers as legibly as you can on the blank sheets of paper provided. Use only **one side** of each sheet. Be sure to number your pages. Put your solution to problem 1 first, and then your solution to number 2, etc.; although, by using enough paper, you can do the problems in any order that suits you. The exam is worth a total of 50 points. SHOW your work. $\boxed{\textit{CIRCLE}}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

I will post the solutions on my website shortly after the exam is finished.

1. (10 points) Find the GENERAL solution of the system of linear equations $Ax = b$. Also, list three SPECIFIC solutions, if possible. CHECK that the specific solutions satisfy the equations.

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 & 2 & 3 \\ 1 & 2 & 3 & 2 & 4 & 6 \\ 1 & 2 & 3 & 3 & 6 & 9 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}, \quad b = \begin{bmatrix} 9 \\ 13 \\ 17 \end{bmatrix}.$$

2. (8 points) Consider the system of linear equations.

$$\begin{aligned} x_1 + ax_2 &= 1 \\ (a-1)x_1 + 6x_2 &= 2. \end{aligned}$$

   (a) Which values for a cause the system to have no solution?
   (b) Which values for a cause the system to have exactly one solution?
   (c) Which values for a cause the system to have an infinite number of solutions?

Explain thoroughly.

3. (8 points) Are the vectors

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}, \quad v_3 = \begin{bmatrix} -1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$$

   linearly independent? **Explain thoroughly.**

4. (8 points) Recall that the matrix A is *symmetric* if $A^{\mathrm{T}} = A$. Let A and B be 2×2 symmetric matrices. Does AB have to be symmetric? If yes, prove your answer. If no, give a counterexample.

5. (8 points) Let v_1, v_2, v_3 be linearly independent vectors in $\mathbb{R}^5$. Let $w_1 = v_1 + v_2 + v_3$, $w_2 = v_1 + v_3$, and $w_3 = v_2 + v_3$. Do the vectors w_1, w_2, w_3 have to be linearly independent? If yes, prove your answer. If no, give a counterexample.

6. (8 points) Let A and B be 2×2 matrices. Does the equation $(A-B)(A+B) = A^2 - B^2$ always hold? If yes, prove your answer. if no, give a counterexample.