Math 544, Exam 3, Fall, 2020

IF IT IS NECESSARY FOR YOU TO LEAVE THE ROOM DURING THE EXAM, PLEASE LET ME KNOW WHEN YOU LEAVE, WHEN YOU RETURN, AND LEAVE YOUR PHONE WITH ME while you are gone.

Write everything on the blank paper that you brought. There should be nothing on your desk except this exam, the blank paper that you brought, and a pen or pencil. When you are finished, put your solutions in order, then send a picture of your solutions to

kustin@math.sc.edu

The exam is worth 50 points. Please make your work coherent, complete, and correct.

Recall that for each non-negative integer d, $\mathcal{P}_d$ is the vector space of all polynomials of degree d or less in one variable with real number coefficients.

(1) Let A be a nonsingular $n \times n$ matrix and B be an $n \times n$ matrix? Answer each question. If the answer is "yes", then prove the statement. If the answer is "no", give an example.

   (a) (10 points) Does the column space of AB have to equal the column space of B?

   (b) (10 points) Does the null space of AB have to equal the null space of B?

   (c) (10 points) Does the rank of AB have to equal the rank of B?

(2) (10 points) Let $V = \{p \in \mathcal{P}_5 \mid p(1) = 0,\ p'(1) = 0,\ \text{and } p''(1) = 0\}$. Give a basis for V. Prove your answer.

(3) (10 points) Let U and V be finite dimensional subspaces of the vector space W. Recall that $U \cap V$ is the vector space

$$U \cap V = \{w \in W \mid w \in U \text{ and } w \in V\}.$$

Let $w_1, \dots, w_r$ be a basis for $U \cap V$. Let $u_1, \dots, u_s$ be in U and $v_1, \dots, v_t$ in V be vectors so that the vectors $w_1, \dots, w_r, u_1, \dots, u_s$ are linearly independent and the vectors $w_1, \dots, w_r, v_1, \dots, v_t$ are linearly independent. Prove that the vectors $w_1, \dots, w_r, u_1, \dots, u_s, v_1, \dots, v_t$ are linearly independent.