Math 544, Exam 3, Summer 2012

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. There are **6** problems. SHOW your work. **No Calculators or Cell phones.** Write your answers as legibly as you can. Make your work be coherent and clear. Write in complete sentences. I will post the solutions on my website shortly after the exam is finished.

1. (8 points) Let A be a 3×4 matrix. Suppose that there is a vector v_0 with the property that every vector v with the property $Av = 0$ is a multiple of v_0. Is it possible to solve $Ax = b$ for all $b \in \mathbb{R}^3$? **Explain thoroughly.**

2. (8 points) Suppose that $W \subseteq V$ are vector spaces and that v_1, v_2, v_3, v_4 is a basis for V. Suppose further, that $v_1 \in W$ but $v_2 \notin W$, $v_3 \notin W$, and $v_4 \notin W$. List all of the possible values for $\dim W$. **Explain thoroughly.**

3. (8 points) Let U and V be subspaces of a vector space W and that $z_1, \ldots, z_r$, $u_1, \ldots, u_s$, and $v_1, \ldots, v_t$ are vectors in W. Soppose further that $z_1, \ldots, z_r$ is a basis for the intersection $U \cap V$ of U and V; $z_1, \ldots, z_r$, $u_1, \ldots, u_s$ is a basis for U and $z_1, \ldots, z_r$, $v_1, \ldots, v_t$ is a basis for V. Prove that the vectors $z_1, \ldots, z_r$, $u_1, \ldots, u_s$, $v_1, \ldots, v_t$ are linearly independent.

4. (8 points) Find a matrix A with AB equal to the identity matrix. You may do the problem anyway you like; in particular, you are welcome to notice that the columns of B form an orthogonal set,
$$B = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & -3 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}.$$

5. (9 points) Let $A = \begin{bmatrix} 1 & 4 & 1 & 5 & 2 & 4 & 6 \\ 1 & 4 & 2 & 10 & 3 & 6 & 9 \\ 1 & 4 & 1 & 5 & 3 & 6 & 9 \\ 3 & 12 & 4 & 20 & 8 & 16 & 24 \end{bmatrix}$. Find a basis for the null space of A. Find a basis for the column space of A. Find a basis for the row space of A. Express each column of A in terms of your basis for the column space. Express each row of A in terms of your basis for the row space. **Check your answer.**

6. (9 points) Find an orthogonal basis for the null space of $A = [1 \quad 2 \quad 1 \quad 3]$. **Check your answer.**