

Math 544, Exam 3, Spring 2011

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. There are **8** problems on **TWO SIDES**. SHOW your work. **No Calculators or Cell phones.** Write your answers as legibly as you can. Make your work be coherent and clear. **Write in complete sentences.** I will post the solutions on my website.

Please Note: The next quiz will be **Thursday**.

Please Note: In this exam, if V is a subset of \mathbb{R}^n for some n , then the phrases: “ V is a subspace of \mathbb{R}^n ” and “ V is a vector space” have exactly the same meaning.

1. (7 points) **Define** “dimension”. Use complete sentences. Include everything that is necessary, but nothing more.
2. (7 points) **Define** “basis”. Use complete sentences. Include everything that is necessary, but nothing more.
3. (6 points) **Define** “subspace of \mathbb{R}^n ”. Use complete sentences. Include everything that is necessary, but nothing more.
4. (6 points) **State** the Four Theorems about Dimension. Use complete sentences. Include everything that is necessary, but nothing more.
5. (6 points) Let

$$V = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \in \mathbb{R}^3 \mid ab = 0 \right\}.$$

Is V a vector space? If yes, prove your answer. If no, give an example which shows why V is not a vector space. **Record a thorough answer. Use complete sentences.**

There are more problems on the other side.

6. (6 points) Let W be a subspace of \mathbb{R}^n and A be an $m \times n$ matrix. Let

$$V = \{Aw \mid w \in W\}.$$

Is V a vector space? If yes, prove your answer. If no, give an example which shows why V is not a vector space. **Record a thorough answer. Use complete sentences.**

7. (6 points) Let A and B be $n \times n$ matrices with B non-singular. Does the column space of BA have to equal the column space of A ? **Prove your answer very thoroughly. Use complete sentences.**
8. (6 points) Let A and B be $n \times n$ matrices with B non-singular. Does the dimension of the column space of BA have to equal the dimension of the column space of A ? **Prove your answer very thoroughly. Use complete sentences.**