Math 544,   Exam 2,   Spring, 2022

You should KEEP this piece of paper. Write everything on the **blank paper provided**. Return the problems **in order** (use as much paper as necessary), use **only one side** of each piece of paper. Number your pages and write your name on each page. Take a picture of your exam (for your records) just before you turn the exam in. I will e-mail your grade and my comments to you. I will keep your exam. **Fold your exam in half** before you turn it in.

The exam is worth 50 points. Problems 1 and 2 are worth 9 points each. Problems 3 – 6 are worth 8 points each. **Make your work coherent, complete, and correct.** Please $\boxed{CIRCLE}$ your answer. Please **CHECK** your answer whenever possible.

The solutions will be posted later today.

No Calculators, Cell phones, computers, notes, etc.

(1) Define "linearly independent". Use complete sentences. Include everything that is necessary, but nothing more.

(2) Define "nonsingular". Use complete sentences. Include everything that is necessary, but nothing more.

(3) Let v_1, v_2, and v_3 be vectors in $\mathbb{R}^n$ and M be a nonsingular $n \times n$ matrix. Suppose the vectors v_1, v_2, v_3 are linearly independent. Do the vectors Mv_1, Mv_2, Mv_3 have to be linearly independent? If yes, prove your answer. If no, give a counterexample.

(4) Let A be a square matrix, v_1 and v_2 be non-zero vectors with $Av_1 = \lambda_1 v_1$ and $Av_2 = \lambda_2 v_2$, where λ_1 and λ_2 are real numbers with $\lambda_1 \neq \lambda_2$. Prove that the vectors v_1, v_2 are linearly independent.

(5) Let a and b be fixed vectors in $\mathbb{R}^3$. Consider

$$W = \left\{ x \in \mathbb{R}^3 \,\middle|\, a^{\mathrm{T}}x = 0 \quad \text{and} \quad b^{\mathrm{T}}x = 0 \right\}.$$

Is the set W a vector space? Explain thoroughly.

(6) Solve the system of equations $Ax = b$, where

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 & 9 \\ 1 & 2 & 3 & 2 & 13 \\ 2 & 4 & 6 & 3 & 22 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 13 \\ 20 \\ 33 \end{bmatrix}.$$

If $Ax = b$ has more than one solution, then give the general solution, four particular solutions, and check that your particular solutions work.