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4. Define “dimension”. Use complete sentences. Include everything that is necessary, but nothing more.

5. Let

$$V = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 \mid 2x_1 + 3x_2 - 4x_3 = 5 \right\}.$$

Is V a vector space? Explain thoroughly.

6. Let $a = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ be fixed elements of \mathbb{R}^3 , and let

$$V = \left\{ x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 \mid a^T x = 0 \text{ and } b^T x = 0 \right\}.$$

Is V a vector space? Explain thoroughly.

7. Let A be the matrix

$$A = \begin{bmatrix} 1 & 3 & 4 & 2 & 4 \\ 1 & 3 & 4 & 3 & 6 \\ 2 & 6 & 8 & 5 & 10 \end{bmatrix}$$

- (a) Find a basis for the null space of A .
- (b) Find a basis for the column space of A .
- (c) Find a basis for the row space of A .
- (d) Write each column of A as a linear combination of your answer to (b).
- (e) Write each row of A as a linear combination of your answer to (c).