Math 544, Exam 2, Summer 2012

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. There are **8** problems. SHOW your work. **No Calculators or Cell phones.** Write your answers as legibly as you can. Make your work be coherent and clear. Write in complete sentences. I will post the solutions on my website shortly after the exam is finished.

1. (6 points) Define "non-singular". Use complete sentences. Include everything that is necessary, but nothing more.

2. (7 points) Let A be an $n \times n$ matrix. List three statements that are equivalent to the statement " A is non-singular". Please do not repeat your answer to problem 1 and only use statements that we have studied thoroughly.

3. (7 points) Define "linearly independent". Use complete sentences. Include everything that is necessary, but nothing more.

4. (6 points) Suppose that v_1, v_2, and v_3 are linearly independent vectors in $\mathbb{R}^n$ and M is an $n \times n$ matrix. Do the vectors Mv_1, Mv_2, Mv_3 have to be linearly independent? If yes, prove your answer. If no, give a counterexample.

5. (6 points) Let v_1, v_2, and v_3 be vectors in $\mathbb{R}^n$ and M be an $n \times n$ matrix. Suppose the vectors Mv_1, Mv_2, Mv_3 are linearly independent. Do the vectors v_1, v_2, v_3 have to be linearly independent? If yes, prove your answer. If no, give a counterexample.

6. (6 points) Suppose that v_1, v_2, and v_3 are linearly independent vectors in $\mathbb{R}^n$ and M is an invertible $n \times n$ matrix. Do the vectors Mv_1, Mv_2, Mv_3 have to be linearly independent? If yes, prove your answer. If no, give a counterexample.

7. (6 points) Let A be the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. What is the inverse of A? Please make sure that your answer is correct.

8. (6 points) Let $V = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \in \mathbb{R}^3 \,\middle|\, ab = 0 \right\}$? Is V a vector space? If yes, prove it. If not, illustrate with an example.