Math 544, Exam 2, Spring 2011

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. There are **9** problems on **TWO SIDES**. SHOW your work. **No Calculators or Cell phones.** Write your answers as legibly as you can. Make your work be coherent and clear. Write in complete sentences. I will post the solutions on my website shortly after the exam is finished.

1. (5 points) **Define** "linearly independent". Use complete sentences. Include everything that is necessary, but nothing more.

2. (5 points) **Define** "non-singular". Use complete sentences. Include everything that is necessary, but nothing more.

3. (5 points) **State** the Non-Singular Matrix Theorem. Your statement should consist of four equivalent conditions. (I will only accept concepts that we have discussed thoroughly in class.) Include everything that is necessary, but nothing more. Use complete sentences.

4. (5 points) Let U and V be subspaces of $\mathbb{R}^n$. Recall that the *intersection* of U and V is the set of all elements of $\mathbb{R}^n$ that are in **both** U and V. Let W be the intersection of U and V. Is W always a vector space? If "yes", then prove that W is a vector space. If "no", then give an example that demonstrates why W is not always a vector space. Use complete sentences. Justify each step.

5. (6 points) Let U and V be subspaces of $\mathbb{R}^n$. Recall that the *union* of U and V is the set of all elements of $\mathbb{R}^n$ that are in **at least one** of the sets U or V. Let W be the union of U and V. Is W always a vector space? If "yes", then prove that W is a vector space. If "no", then give an example that demonstrates why W is not always a vector space. Use complete sentences. Justify each step.

There are more problems on the other side.

6. (6 points) Let M be an $m \times n$ matrix. Let $W = \{v \in \mathbb{R}^n \mid Mv = 0\}$. Is W always a vector space? If "yes", then prove that W is a vector space. If "no", then give an example that demonstrates why W is not always a vector space. Use complete sentences. Justify each step.

7. (6 points) Let A and B be $n \times n$ matrices. How is $(AB)^{\mathrm{T}}$ related to the product of A^{T} and B^{T}? Prove that your answer is correct. For full credit your answer should work for all n. Justify each step. Your notation must make sense. Use complete sentences.

8. (6 points) Let A and B be $n \times n$ matrices with AB non-singular. Prove that A and B are both non-singular. Use complete sentences. Justify each step.

9. (6 points) Let v_1, v_2, v_3 be vectors in $\mathbb{R}^4$ with
   (a) v_1, v_2 a linearly independent set of vectors,
   (b) v_1, v_3 a linearly independent set of vectors, and
   (c) v_2, v_3 a linearly independent set of vectors.

Does v_1, v_2, v_3 have to be a linearly independent set of vectors ? If yes, prove your answer. If no, give a counterexample. Use complete sentences.