

Math 544, Exam 1, Spring, 2022

**You should KEEP this piece of paper.** Write everything on the **blank paper provided**. Return the problems **in order** (use as much paper as necessary), use **only one side** of each piece of paper. Number your pages and write your name on each page. Take a picture of your exam (for your records) just before you turn the exam in. I will e-mail your grade and my comments to you. I will keep your exam. **Fold your exam in half** before you turn it in.

The exam is worth 50 points. Each problem is worth 10 points. **Make your work coherent, complete, and correct.** Please CIRCLE your answer. Please **CHECK** your answer whenever possible.

The solutions will be posted later today.

**No Calculators, Cell phones, computers, notes, etc.**

(1) Solve the system of equations  $Ax = b$ , where

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 & 9 \\ 1 & 2 & 3 & 2 & 13 \\ 2 & 4 & 6 & 3 & 22 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 13 \\ 20 \\ 33 \end{bmatrix}.$$

If  $Ax = b$  has more than one solution, then give the general solution and four particular solutions.

(2) Consider the system of equations  $Ax = b$ , where

$$A = \begin{bmatrix} 1 & a^2 - 1 \\ 2 & a^2 - 1 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 3 - a \\ 5 - a \end{bmatrix}.$$

- (a) Find all values of  $a$  for which the system of equations has no solution.
  - (b) Find all values of  $a$  for which the system of equations has exactly one solution.
  - (c) Find all values of  $a$  for which the system of equations has an infinite number of solutions.
- (3) Define “linearly independent”. Use complete sentences. Include everything that is necessary, but nothing more.
- (4) Suppose  $v_1$ ,  $v_2$ , and  $v_3$  are three vectors in  $\mathbb{R}^m$ , for some  $m$ , with  $v_1, v_2$  linearly independent,  $v_1, v_3$  linearly independent, and  $v_2, v_3$  linearly independent. Do the vectors  $v_1, v_2, v_3$  have to be linearly independent? If the answer is yes, prove it. If the answer is no, give a counterexample.
- (5) Let  $v_1$ ,  $v_2$ , and  $v_3$  be non-zero vectors in  $\mathbb{R}^m$ , for some  $m$ . Suppose that  $v_i^T v_j = 0$  for all subscripts  $i$  and  $j$  with  $i \neq j$ . Do the vectors  $v_1, v_2, v_3$  have to be linearly independent? If the answer is yes, prove it. If the answer is no, give a counterexample.