Math 544,   Exam 1,   Spring 2016

Write everything on the blank paper provided. **You should KEEP this piece of paper.** If possible: return the problems in order (use as much paper as necessary), use only one side of each piece of paper, and leave 1 square inch in the upper left hand corner for the staple. If you forget some of these requests, don't worry about it – I will still grade your exam.

The exam is worth 50 points. Each problem is worth 10 points. SHOW your work. **No Calculators or Cell phones.** Write your answers as legibly as you can. Make your work be coherent and clear. Write in complete sentences. I will post the solutions on my website shortly after the exam is finished.

1. Find the GENERAL solution of the system of linear equations $Ax = b$. Also, list three SPECIFIC solutions, if possible. CHECK that the specific solutions satisfy the equations. $\boxed{CIRCLE}$ your answer.

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 & 0 & 0 \\ 1 & 2 & 3 & 1 & 4 & 0 \\ 2 & 4 & 6 & 1 & 4 & 1 \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}, \quad b = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}.$$

2. **Define** "linearly independent". Use complete sentences. Include everything that is necessary, but nothing more.

3. Consider the system of equations $Ax = b$ where $A = \begin{bmatrix} 1 & -a \\ a & -1 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, and $b = \begin{bmatrix} 3 \\ 4a - 1 \end{bmatrix}$.

   (a) For which values of a does the system of equations have no solution?
   (b) For which values of a does the system of equations have exactly one solution?
   (c) For which values of a does the system of equations have more than sone solution?

4. Let v_1, v_2, v_3, v_4 be linearly independent vectors in $\mathbb{R}^m$, for some m. Do the vectors v_1, v_2, v_3 have to be linearly independent? If yes, prove the statement. If no give an example.

5. Let v_1, v_2, v_3, v_4 be linearly dependent vectors in $\mathbb{R}^m$, for some m. Do the vectors v_1, v_2, v_3 have to be linearly dependent? If yes, prove the statement. If no give an example.