PRINT Your Name:________________________

Math 544, Exam 1, Fall 2009

Write your answers as legibly as you can.

Make your work be coherent and clear. Write in complete sentences

There are 12 problems on 6 pages. Problems 1, 2, 3, 4 are worth 9 points each. Each of the other problems is worth problem is worth 8 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators. No phones.**

I will post the solutions on my website shortly after the exam is finished.

1. Find the GENERAL solution of the following system of linear equations. Also, list three SPECIFIC solutions, if possible. CHECK that the specific solutions satisfy the equations.

$$\begin{array}{rcl} x_1+x_2 \qquad\qquad - x_5 &=& 1 \\ x_2+2x_3+x_4+3x_5 &=& 1 \\ x_1 \qquad - x_3+x_4+x_5 &=& 0 \end{array}$$

2. Define “linearly independent”. Use complete sentences. Include everything that is necessary, but nothing more.

3.
   (a) Define “non-singular”. Use complete sentences. Include everything that is necessary, but nothing more.
   (b) Let A be an $n \times n$ matrix. List three statements that are equivalent to the statement “A is non-singular”.

4. Let A and B be 2×2 matrices with A not equal to the zero matrix and $A^2 = AB$. Does A have to equal B? If yes, prove your answer. If no, give a counterexample.

5. Let A and B be 2×2 matrices. Does $(A+B)^2$ have to equal $A^2+2AB+B^2$? If yes, prove your answer. If no, give a counterexample.

6. Recall that the matrix A is called symmetric if $A^{\mathrm{T}} = A$. Let A and B be 2×2 symmetric matrices. Does AB have to be a symmetric matrix? If yes, prove your answer. If no, give a counterexample.

7. Let v_1, v_2, v_3, v_4 be vectors in $\mathbb{R}^4$ with v_1, v_2, v_3 linearly dependent. Do v_1, v_2, v_3, v_4 have to be linearly dependent? If yes, prove your answer. If no, give a counterexample.

8. Let v_1, v_2, v_3, v_4 be vectors in $\mathbb{R}^4$ with v_1, v_2, v_3 linearly independent. Do v_1, v_2, v_3, v_4 have to be linearly independent? If yes, prove your answer. If no, give a counterexample.

9. Suppose v_1 , v_2 and v_3 are vectors in $\mathbb{R}^4$ with v_1, v_2 linearly independent, v_1, v_3 linearly independent, and v_2, v_3 linearly independent. Do the vectors v_1, v_2, v_3 have to be linearly independent? If yes, give a proof. If no, give an example.

10. Suppose A is a 2×3 matrix and B is a 3×2 matrix with $AB = I$. Does BA have to equal I ? If yes, give a proof. If no, give an example.

11. Suppose A and B are 2×2 matrices with $AB = I$. Does BA have to equal I? If yes, give a proof. If no, give an example.

12. Suppose v_1, v_2, v_3 are linearly independent vectors in $\mathbb{R}^4$. Do the vectors $v_1 + v_2$, $v_2 - v_3$, $v_3 + v_1$ have to be linearly independent? If yes, give a proof. If no, give an example.