MATH 544, 1997, EXAM 4

PRINT Your Name:__________________________________

There are 10 problems on 6 pages. Each problem is worth 10 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **NO CALCULATORS.**

1. Define "linear transformation".

2. Define "eigenvalue".

3. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If A and B are 2×2 matrices, then $\det(A+B) = \det A + \det B$.

4. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If A is a 2×2 matrix and c is a constant, then $\det(cA) = c \det A$.

5. Consider the linear transformation $T\colon \mathbb{R}^2 \to \mathbb{R}^3$ with

$$T\left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}.$$

Find a matrix A with $T(v) = Av$ for all $v \in \mathbb{R}^2$.

6. Solve $Ax = b$, where

$$A = \begin{bmatrix} 10 & 1 & 35 \\ 11 & -2 & 2 \\ 12 & 1 & -31 \end{bmatrix}, \quad \text{and} \quad b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

(Note. The columns of A form an orthogonal set.)

7. Find all eigenvalues and eigenvectors of

$$A = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix}.$$

8. Consider the function $T\colon \mathbb{R}^2 \to \mathbb{R}^2$, which is given by reflection across the line $y = x + 1$. Is T a linear transformation? If so, then give a matrix with $T(v) = Av$ for all $v \in \mathbb{R}^2$. If not, then show why not.

9. Consider the function $T\colon \mathbb{R}^2 \to \mathbb{R}^2$, which is given by reflection across the line $y = -x$. Is T a linear transformation? If so, then give a matrix with $T(v) = Av$ for all $v \in \mathbb{R}^2$. If not, then show why not.

10. Find an orthogonal set which is a basis for the null space of $\begin{bmatrix} 1 & 2 & 2 & 1 \end{bmatrix}$.