MATH 544, 1997, EXAM 2

PRINT Your Name:________________________________

There are 10 problems on 4 pages. Each problem is worth 10 points. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible.

1. Define "linearly independent".

2. Define "nonsingular".

3. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If A and B are $n \times n$ matrices, then

$$\text{the null space of } A + B \subseteq \text{the null space of } A \cap \text{the null space of } B .$$

4. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If V and W are subspaces of $\mathbb{R}^n$, then the union $V \cup W$ is a subspace of $\mathbb{R}^n$.

5. True or False. If the statement is true, then PROVE the statement. If the statement is false, then give a COUNTEREXAMPLE. If V and W are subspaces of $\mathbb{R}^n$, then the intersection $V \cap W$ is a subspace of $\mathbb{R}^n$.

6. Let $V = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \in \mathbb{R}^2 \;\middle|\; x^2 + y^2 = 1 \right\}$. Is V a subspace of $\mathbb{R}^2$? Justify your answer.

7. Is $v = \begin{bmatrix} 0 \\ 3 \\ 5 \end{bmatrix}$ in the column space of $A = \begin{bmatrix} 1 & 2 \\ 4 & 5 \\ 6 & 7 \end{bmatrix}$? Justify your answer.

8. Let A be a fixed 2×3 matrix. Let $V = \{ v \in \mathbb{R}^2 \mid v = Ax \text{ for some } x \in \mathbb{R}^3 \}$. Is V a subspace of $\mathbb{R}^2$? Justify your answer.

Problems 9 and 10 both use the matrix

$$A = \begin{bmatrix} 1 & 3 & 0 & 2 & 0 \\ 1 & 3 & 1 & 6 & 0 \\ 1 & 3 & 1 & 6 & 1 \\ 2 & 6 & 1 & 8 & 1 \end{bmatrix}.$$

9. Find a basis for the null space of A.

10. Find a basis for the column space of A.