Math 544, Fall 2009, Exam 3

Use my paper. **Please turn the problems in order. Please leave 1 square inch in the upper left hand corner for the staple.**

The exam is worth 50 points. There are 8 problems. SHOW your work. $\boxed{CIRCLE}$ your answer. **CHECK** your answer whenever possible. **No Calculators.**

1. (6 points) Define "linear transformation". Use complete sentences.

2. (6 points) The *trace* of the square matrix A is the sum of the numbers on its main diagonal. Let V be the set of all 3×3 matrices with trace 0. Is the set V a vector space? Explain.

3. (6 points) Let $A = \begin{bmatrix} 7 & 6 \\ -3 & -2 \end{bmatrix}$. Find a matrix B with $B^2 = A$. **Check your answer.**

4. (6 points) Find an orthogonal set which is a basis for the null space of $A = [\,1 \quad -1 \quad 1 \quad -1\,]$. **Check your answer.**

5. Let $T\colon V \to W$ be a linear transformation of vector spaces. Suppose that the vectors $v_1 \dots, v_a$ in V are a basis for the null space of T and that $w_1, \dots, w_b$ in W are a basis for the image of T. Let $u_1, \dots, u_b$ in V be vectors with $T(u_i) = w_i$ for $1 \le i \le b$.
   (a) (4 points) Prove that $v_1, \dots, v_a, u_1, \dots, u_b$ span V.
   (b) (4 points) Prove that $v_1, \dots, v_a, u_1, \dots, u_b$ are linearly independent vectors in V.

6. (6 points) Let

$$A = \begin{bmatrix} 1 & 1 & 0 & -1 \\ 1 & -1 & 0 & -1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & -1 & 1 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}.$$

   Solve $Ax = b$. (You may do the problem any way you wish; however, you may find it helpful to notice that the columns of A form an orthogonal set.) **Check your answer.**

7. (6 points) Let $W = \{f\colon \mathbb{R} \to \mathbb{R} \mid f \text{ is differentiable}\}$. Is W a vector space? Explain.

8. (6 points) Let $T\colon \mathbb{R}^2 \to \mathbb{R}^2$ be reflection across the line $y = \frac{1}{\sqrt{3}}x$. Find a matrix M with $T(v) = Mv$ for all vectors $v \in \mathbb{R}^2$.