Quiz for January 21, 2010

The quiz is worth 5 points. Remove EVERYTHING from your desk except this quiz and a pen or pencil. SHOW your work. Express your work in a neat an coherent manner. BOX your answer.

Suppose that a car skids 15 meters if it is moving at 50 km/hr when the brakes are applied. Assuming that the car has the same constant deceleration, how far will it skid if it is moving at 100 km/hr when the brakes are applied?

ANSWER: Let x(t) be the position of the car at time t. Measure t in hours and x in km. We take x(0) = 0. We are told that x''(t) = -k for some positive constant k. For the first car, we have x'(0) = 50 and x(when the car stops) = .015. For the second car, we have x'(0) = 100 and we are supposed to find x(when the car stops).

For the first car: x' = -kt + 50 and $x = -kt^2/2 + 50t$. The car stops when x'(t) = 0; so, t = 50/k and

.015 =
$$x$$
(when the car stops) = $x(50/k) = -k(50/k)^2/2 + 50(50/k)$
= $\frac{-(50)^2/2 + (50)^2}{k} = \frac{50^2}{2k}$.

We have learned that $k = (50)^2/2(.015)$.

Now we consider car 2. We have x' = -kt + 100 and $x = -kt^2/2 + 100t$. The car stops when x'(t) = 0; so, t = 100/k and

$$x$$
(when the car stops) = $x(100/k) = -k(100/k)^2/2 + 100(100/k) = \frac{100^2}{2k}$
= $\frac{100^2}{2} \frac{2(.015)}{(50)^2} = 4(.015) = .06 \text{ km} = \boxed{60 \text{ meters.}}$