PRINT Your Name:________________________________

Quiz for February 4, 2010

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil. SHOW every step**. Express your work in a neat and coherent manner. $\boxed{\text{BOX}}$ your answer.

Derive the solution of $P' = kP(P-M)$, $P(0) = P_0$, where k and M are positive constants.

ANSWER: We solve $\frac{dP}{P(P-M)} = kdt$. We apply the technique of partial fractions. Write $\frac{A}{P} + \frac{B}{P-M} = \frac{1}{P(P-M)}$. Multiply both sides by $P(P-M)$ to get: $A(P-M)+BP = 1$. Plug in $P = 0$ to see that $A = \frac{-1}{M}$. Plug in $P = M$ to see $B = \frac{1}{M}$. We must solve

$$\frac{1}{M}\left(\frac{1}{P-M} - \frac{1}{P}\right) dP = kdt.$$

Integrate both sides:

$$\frac{1}{M} \ln\left(\frac{P-M}{P}\right) = kt + C.$$

Multiply both sides by M :

$$\ln\left(\frac{P-M}{P}\right) = Mkt + MC.$$

Exponentiate:

$$\frac{P-M}{P} = Ke^{Mkt},$$

where K is the constant e^{MC} . This is a good place to evaluate K . Plug $t = 0$ in to the equation to see that $\frac{P_0-M}{P_0} = K$. Multiply both sides by P

$$P - M = PKe^{Mkt}.$$

Subtract PKe^{Mkt} from both sides and add M to both sides:

$$P - PKe^{Mkt} = M.$$

Factor out P :

$$P(1 - Ke^{Mkt}) = M.$$

Divide:

$$P = \frac{M}{1 - Ke^{Mkt}}.$$

Drop $\frac{P_0 - M}{P_0}$ in for K :

$$P = \frac{M}{1 - \left(\frac{P_0 - M}{P_0}\right) e^{Mkt}}.$$

Multiply the top and bottom by P_0 :

$$P = \frac{MP_0}{P_0 - (P_0 - M)e^{Mkt}}.$$

Thus,

$$\boxed{P = \frac{MP_0}{P_0 + (M - P_0)e^{Mkt}}.}$$