PRINT Your Name:____________________________________

Quiz for February 23, 2010

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil. SHOW every step**. Express your work in a neat and coherent manner. $\boxed{\text{BOX}}$ your answer.

Solve $(x^2+1)\frac{dy}{dx}+3xy=6x$. Express your answer in the form $y(x)$. **Check your answer.**

ANSWER: This is a first order linear DE:

$$\frac{dy}{dx}+\frac{3x}{x^2+1}y=\frac{6x}{x^2+1}.$$

The integrating factor is

$$e^{\int \frac{3x}{x^2+1}dx}=e^{3/2\ln(x^2+1)}=(x^2+1)^{3/2}.$$

Multiply both sides by the integrating factor:

$$(x^2+1)^{3/2}\frac{dy}{dx}+3x(x^2+1)^{1/2}y=6x(x^2+1)^{1/2}.$$

$$\frac{d}{dx}((x^2+1)^{3/2}y)=6x(x^2+1)^{1/2}.$$

Integrate both sides:

$$(x^2+1)^{3/2}y=2(x^2+1)^{3/2}+C$$

$$\boxed{y=2+C(x^2+1)^{-3/2}}$$

Check. We see that $dy/dx=-(3/2)C(x^2+1)^{-5/2}2x=\frac{-3Cx}{(x^2+1)^{5/2}}$. So,

$$(x^2+1)\frac{dy}{dx}+3xy=(x^2+1)\frac{-3Cx}{(x^2+1)^{5/2}}+3x(2+C(x^2+1)^{-3/2})=6x \checkmark$$