PRINT Your Name:__________________________________

Quiz for February 11, 2010

The quiz is worth 5 points. **Remove EVERYTHING from your desk except this quiz and a pen or pencil. SHOW every step**. Express your work in a neat and coherent manner. $\boxed{\text{BOX}}$ your answer.

Suppose that a car starts from rest, its engine providing an acceleration of 10 f/s^2, while air resistance provides .1 f/s^2 of deceleration for each foot per second of the car's velocity.

(a) Find the car's maximum possible (limiting) velocity.

(b) Find how long it takes the car to attain 90% of its velocity and how far it travels while doing so.

ANSWER: We solve the initial value problem: $\frac{dv}{dt} = 10 - (1/10)v$, $v(0) = 0$. We have

$$\frac{dv}{10 - v/10} = dt.$$

Integrate both sides to obtain

$$-10 \ln |10 - v/10| = t + C.$$

Divide both sides by -10 to obtain

$$\ln |10 - v/10| = -(t/10) - (1/10)C.$$

Exponentiate to see

$$10 - v/10 = Ke^{-t/10},$$

where $K = \pm e^{-(1/10)C}$. Plug in $t = 0$ to learn that $K = 10$. Multiply by 10:

$$100 - v = 100e^{-t/10},$$

which yields

$$100 - 100e^{-t/10} = v.$$

(a) We see that $\lim_{t\to\infty} v = \boxed{100\text{f/s}}$

(b) We see that $v(t) = 90\%(100)$ when

$$90 = 100 - 100e^{-t/10}.$$

We solve for t:

$$100e^{-t/10} = 10$$

$$e^{-t/10} = 1/10$$

$$-t/10 = \ln(1/10).$$

Of course, we know $\ln(1/10) = -\ln 10$; so the answer to the first part of (b) is $\boxed{t = 10 \ln 10 \text{ s}}$.

We now find the answer to the second part of (b). Integrate $100 - 100e^{-t/10} = v$ to learn

$$x = 100t + 1000e^{-t/10} + C_0.$$

The distance travel by the car from $t = 0$ until $t = 10 \ln 10$ is

$$x(10 \ln 10) - x(0) = 1000 \ln 10 + 1000e^{-\ln 10} - 1000 = \boxed{[1000(\ln 10 - 1) + 100] f}.$$