$$\frac{d}{dx}(v/x^2) = -1/x^2.$$

Integrate both sides

$$v/x^2 = 1/x + C$$
$$v = x + Cx^2$$
$$y^{-1/3} = x + Cx^2$$
$$\boxed{\frac{1}{(x+Cx^2)^3} = y}$$

Check. We see that $dy/dx = (-3)(x+Cx^2)^{-4}(1+2Cx)$. So

$$x(dy/dx) + 6y = \frac{-3(x+2Cx^2)}{(x+Cx^2)^4} + \frac{6}{(x+Cx^2)^3}$$

$$= \frac{-3(x+2Cx^2)}{(x+Cx^2)^4} + \frac{6(x+Cx^2)}{(x+Cx^2)^4} = \frac{3x}{(x+Cx^2)^4} = 3xy^{4/3}. \checkmark$$

5. **Solve** $(x^2+1)\frac{dy}{dx} + 3xy = 6x$. **Express your answer in the form** $y(x)$. **Check your answer.**

This is a first order linear DE:

$$\frac{dy}{dx} + \frac{3x}{x^2+1}y = \frac{6x}{x^2+1}.$$

The integrating factor is

$$e^{\int \frac{3x}{x^2+1}dx} = e^{3/2\ln(x^2+1)} = (x^2+1)^{3/2}.$$

Multiply both sides by the integrating factor:

$$(x^2+1)^{3/2}\frac{dy}{dx} + 3x(x^2+1)^{1/2}y = 6x(x^2+1)^{1/2}.$$

$$\frac{d}{dx}((x^2+1)^{3/2}y) = 6x(x^2+1)^{1/2}.$$

Integrate both sides:

$$(x^2+1)^{3/2}y = 2(x^2+1)^{3/2} + C$$

$$\boxed{y = 2 + C(x^2+1)^{-3/2}}$$

Check. We see that $dy/dx = -(3/2)C(x^2+1)^{-5/2}2x = \frac{-3Cx}{(x^2+1)^{5/2}}$. So,

$$(x^2+1)\frac{dy}{dx} + 3xy = (x^2+1)\frac{-3Cx}{(x^2+1)^{5/2}} + 3x(2 + C(x^2+1)^{-3/2}) = 6x \checkmark$$