

We see that $(x^2 + y^2)|_{(0,1)} = 1$; so, the segment from $(0, 1)$ to $(1/10, y_1)$ has slope equal to 1. This segment lives on the line

$$y - 1 = 1(x - 0),$$

which is $y = x + 1$. Thus, $y_1 = 1/10 + 1 = 11/10$.

The line segment from $(1/10, y_1)$ to $(2/10, y_2)$ has slope equal to

$$(x^2 + y^2)|_{1/10, y_1} = (x^2 + y^2)|_{(1/10, 11/10)} = (1/10)^2 + (11/10)^2 = 122/100.$$

This segment lives on the line

$$y - 11/10 = (122/100)(x - 1/10)$$

which is

$$y = (122/100)x + (1100 - 122)/1000$$

Our approximation of $y(2/10)$ is $y_2 = (122/100)(2/10) + (1100 - 122)/1000 = 1.222$.

4. **Solve $x \frac{dy}{dx} + 6y = 3xy^{4/3}$. Express your answer in the form $y(x)$. Check your answer.**

This is a Bernoulli equation. Let $v = y^{-1/3}$. Thus, $v' = (-1/3)y^{-4/3}y'$ and $-3v'y^{4/3} = y'$. The original DE is

$$x(-3v'y^{4/3}) + 6y = 3xy^{4/3}.$$

Divide by $y^{4/3}$ to get

$$-3xv' + 6y^{-1/3} = 3x,$$

which is the First Order Linear DE

$$-3xv' + 6v = 3x.$$

Divide by $-3x$:

$$v' - (2/x)v = -1.$$

The integrating factor is

$$\mu = e^{\int -(2/x)dx} = e^{-2 \ln x} = 1/x^2.$$

Multiply both sides by the integrating factor

$$v'/x^2 - 2/x^3 = -1/x^2,$$