

$$\frac{4}{\left(-v_0^{-1/2}(kt\sqrt{v_0} + 2)\right)^2} = v$$

$$\frac{4v_0}{(kt\sqrt{v_0} + 2)^2} = v$$

Integrate again and learn that

$$x = \frac{1}{k\sqrt{v_0}} \frac{-4v_0}{(kt\sqrt{v_0} + 2)} + C_1$$

$$x = \frac{-4\sqrt{v_0}}{k(kt\sqrt{v_0} + 2)} + C_1.$$

Plug in $t = 0$ to learn

$$x_0 = \frac{-4\sqrt{v_0}}{2k} + C_1.$$

So,

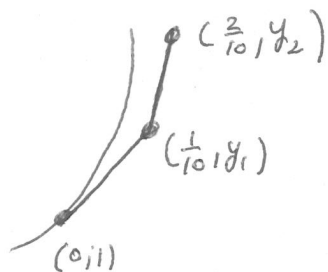
$$x = \frac{-4\sqrt{v_0}}{k(kt\sqrt{v_0} + 2)} + x_0 + \frac{2\sqrt{v_0}}{k}.$$

We compute

$$\lim_{t \rightarrow \infty} x(t) = \lim_{t \rightarrow \infty} \frac{-4\sqrt{v_0}}{k(kt\sqrt{v_0} + 2)} + x_0 + \frac{2\sqrt{v_0}}{k} = \boxed{x_0 + \frac{2\sqrt{v_0}}{k}}.$$

3. Consider the initial value problem $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$. Use Euler's method to approximate $y(2/10)$. Use two steps, each of size $1/10$.

Consider the picture:



The curve represents the correct solution of the initial value problem. We approximate the real solution with the two line segments. The line segment from $(0, 1)$ to $(1/10, y_1)$ has slope equal to $(x^2 + y^2)|_{(0,1)}$. The line segment from $(1/10, y_1)$ to $(2/10, y_2)$ has slope equal to $(x^2 + y^2)|_{(1/10, y_1)}$. Of course, y_2 is our approximation of $y(2/10)$.