Spring 1994 Exam 3 Math 242

① $y'' + y' - 6y = e^{2x}$ (NHDE)

$y'' + y' - 6y = 0$ (HDE)

plug $y = e^{rx}$ into (HDE)

$r^2 + r - 6 = 0 \quad (r-2)(r+3) = 0$ so sol of NHDE is $y = c_1 e^{2x} + c_2 e^{-3x}$

Try $y = Axe^{2x}$ in (NHDE) $\quad y' = 2Axe^{2x} + Ae^{2x}$

$y'' = 4Axe^{2x} + 2Ae^{2x} + 2Ae^{2x}$

$\cancel{4Axe^{2x}} + 4Ae^{2x} + \cancel{2Axe^{2x}} + Ae^{2x} - \cancel{6Axe^{2x}} = e^{2x}$

$\therefore A = \frac{1}{5}$

$$\boxed{y = c_1 e^{2x} + c_2 e^{-3x} + \tfrac{1}{5} x e^{2x}}$$

② y is missing

The Problem is First order linear in y' $\quad y'' + \frac{1}{x} y' = 4 \quad \mu = e^{\int \frac{1}{x}} = x$

$xy' = \int 4x\,dx = 2x^2 + c$

$y' = 2x + \frac{c}{x}$

$$\boxed{y = x^2 + c \ln x + k}$$

③ $x^2 y'' + xy' - 4y = x^2$ (NHDE)

$x^2 y'' + xy' - 4y = 0$ (HDE)

HDE is an Euler–Cauchy Equation

Try $y = x^r \quad r(r-1) + r - 4 = 0 \quad r = 2, -2$

the solution of (HDE) is $y = c_1 x^2 + c_2 x^{-2}$

use V of P for NHDE:

$y_{partic} = u_1 x^2 + u_2 x^{-2}$ where $\begin{bmatrix} x^2 & x^{-2} \\ 2x & -2x^{-3} \end{bmatrix} \begin{bmatrix} u_1' \\ u_2' \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ Note!

$\begin{bmatrix} u_1' \\ u_2' \end{bmatrix} = -\frac{x}{4} \begin{bmatrix} -2x^{-3} & -x^{-2} \\ -2x & x^2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} +\frac{x^{-1}}{4} \\ -\frac{x^3}{4} \end{bmatrix} \qquad u_1 = \frac{1}{4} \ln x \qquad u_2 = -\frac{x^4}{16}$

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$y_{partic} = \frac{1}{4} x^2 \ln x - \frac{x^2}{16}$

$$\boxed{y = c_1 x^2 + c_2 x^{-2} + \tfrac{1}{4} x^2 \ln x}$$

4) $y_2 = x \int \frac{-e^{\int \frac{-3x^2}{1+x^3} dx}}{x^2} dx = x \int \frac{1+x^3}{x^2} dx$

$= x \left[-\frac{1}{x} + \frac{x^2}{2} \right] = -1 + \frac{x^3}{2}$

$\boxed{y = c_1 x + c_2 \left(-1 + \frac{x^3}{2} \right)}$

5) The gen solution of $y'' + 9y = e^x$ is

$y = c_1 \cos 3x + c_2 \sin 3x + \frac{1}{10} e^x$

$y' = -3c_1 \sin 3x + 3c_2 \cos 3x + \frac{1}{10} e^x$

$1 = y(0) = c_1 + \frac{1}{10} \qquad c_1 = \frac{9}{10}$

$1 = y'(0) = 3c_2 + \frac{1}{10} \qquad c_2 = \frac{3}{10}$

$y = \frac{9}{10} \cos 3x + \frac{3}{10} \sin 3x + \frac{1}{10} e^x$