Math 242, Sp. 1994 Exam 2

1. Consider the initial value problem

$y' = f(x,y) \quad y(a) = b \quad$ (IVP).

(a) If f is continuous on some rectangle which contains (a,b) in its interior, then (IVP) has a solution which is defined on some interval about $x = a$.

(b) If $\frac{\partial f}{\partial y}$ is continuous on some rectangle which contains (a,b) in its interior, then (IVP) has a unique solution which is defined on some interval about $x = a$.

2. $y = xe^x - e^x + 1$

3. $y' + \frac{2}{x}y = 3$

$\mu = e^{\int \frac{2}{x}} = x^2$

$x^2 y' + 2xy = 3x^2$

$x^2 y = x^3 + C$

$y = x + \frac{C}{x^2}$

$5 = y(1) = 1 + \frac{C}{1}$

$C = 4$

$y = x + \frac{4}{x^2}$

4) $y' = \frac{x+y}{x-y}$

$y' = \frac{1 + \frac{y}{x}}{1 - \frac{y}{x}}$

$v = \frac{y}{x} \qquad xv = y$

$xv' + v = y'$

$xv' + v = \frac{1+v}{1-v}$

$xv' = \frac{1+v - v + v^2}{1-v} = \frac{1+v^2}{1-v}$

$\int \frac{1-v}{1+v^2} = \int \frac{dx}{x}$

$\arctan v - \frac{1}{2}\ln(1+v^2) = \ln x + C$

$\arctan \frac{y}{x} - \frac{1}{2}\ln\left(1 + \frac{y^2}{x^2}\right) = \ln x + C$

5. $y' = y + y^3$

$v = y^{-2}$

$v' = -2y^{-3}y'$

$-2y^{-3}y' = -2y^{-2} - 2$

$v' + 2v = -2$

$e^{2x}v' + 2e^{2x}v = -2e^{2x}$

$e^{2x}v = -e^{2x} + C$

$v = -1 + Ce^{-2x}$

$y^{-2} = (-1 + Ce^{-2x})$

$y = \frac{1}{\sqrt{Ce^{-2x} - 1}}$

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$y = \frac{1}{\;}$

6. $y' = \sqrt{x+y}$

$v = x+y$

$v' = 1 + y'$

$v' - 1 = \sqrt{v}$

$v' = \sqrt{v} + 1$

$\int \frac{dv}{\sqrt{v}+1} = \int dx$

Let $u = \sqrt{v}$

$du = \frac{1}{2\sqrt{v}}\,dv$

$2u\,du = dv$

$\int \frac{2u\,du}{u+1} = \int dx$

$\int 2 - \frac{2}{u+1}\,du = \int dx$

$2u - 2\ln(u+1) = x + c$

$2\sqrt{x+y} - 2\ln(\sqrt{x+y}+1) = x + c$

7. $r^2 + r - 6 = 0$

$(r+3)(r-2)$

$r = -3, 2$

$y = c_1 e^{-3x} + c_2 e^{2x}$

8. $r^2 + 6r + 9 = 0$

$(r+3)^2 = 0$

$r = -3, -3$

$y = c_1 e^{-3x} + c_2 x e^{-3x}$

9. $r^3 - r^2 + 9r - 9 = 0$

$(r-1)(r^2+9) = 0$

$r = 1, 3i, -3i$

$y = c_1 e^x + c_2 \cos 3x + c_3 \sin 3x$

10. $r^2 + 2r + 5 = 0$

$r = \frac{-2 \pm \sqrt{4-20}}{2}$

$r = \frac{-2 \pm 4i}{2} = -1 \pm 2i$

$x = e^{-t}(c_1 \cos 2t + c_2 \sin 2t)$

$3 = x(0) = c_1$

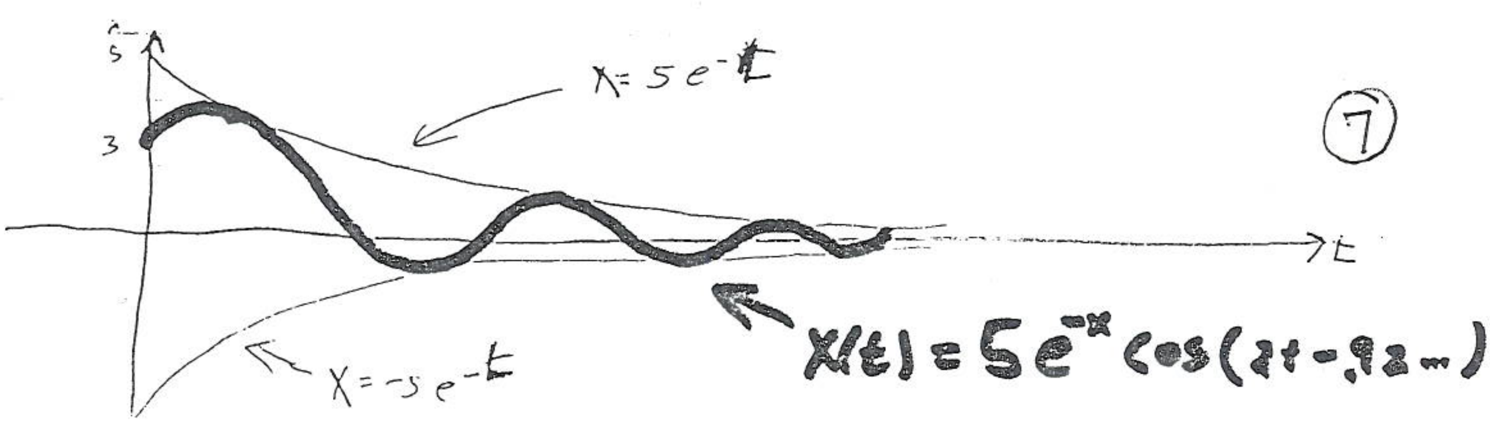
$x' = e^{-t}(-2c_1 \sin 2t + 2c_2 \cos 2t) - e^{-t}(c_1 \cos 2t + c_2 \sin 2t)$

$5 = x'(0) = 2c_2 - 3 \qquad c_2 = 4$

(a) $x(t) = e^{-t}(3\cos 2t + 4\sin 2t) = 5e^{-t}\left(\frac{3}{5}\cos 2t + \frac{4}{5}\sin 2t\right)$

$\cos .9272952 = \frac{3}{5}$

$x(t) = 5e^{-t}\cos(2t - .9272952)$



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