242 1993 Exam 1

① $y'' = -6x^2$

$y' = -2x^3 + c \qquad 3 = y'(1) = -2 + c \qquad 5 = c$

$y' = -2x^3 + 5$

$y = -\frac{1}{2}x^4 + 5x + k \qquad 2 = y(1) = -\frac{1}{2} + 5 + k \qquad -\frac{5}{2} = k$

$y = -\frac{1}{2}x^4 + 5x - \frac{5}{2}$

② $y' + \frac{3}{x}y = 3x^{-\frac{5}{2}}$

$\mu = e^{\int \frac{3}{x}dx} = x^3$

$x^3y' + 3x^2y = 3x^{\frac{1}{2}}$

$x^3y = \int 3x^{\frac{1}{2}}dx = 2x^{\frac{3}{2}} + c$

$y = 2x^{-\frac{3}{2}} + cx^{-3}$

③ $y' = \frac{y}{x} + 2\sqrt{\frac{y}{x}} \qquad v = \frac{y}{x} \qquad xv = y \qquad xv' + v = y'$

$xv' + \cancel{v} = \cancel{v} + 2\sqrt{v}$

$\int v^{-\frac{1}{2}}dv = \int \frac{2}{x}dx$

$2v^{\frac{1}{2}} = 2\ln|x| + c$

$v = (\ln|x| + 2c)^2$

$y = x(\ln|x| + k)^2$

④ $y' = \frac{(1+y)^2}{1+x^2} \qquad y' = f(x,y) \text{ for } f = \frac{(1+y)^2}{1+x^2}$

f is continuous everywhere. $f_y = \frac{2(1+y)}{1+x^2}$ is continuous everywhere.

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(a) (*) has a unique solution on some interval containing $x=a$ <u>for</u> <u>all</u> a and b.

(b) $\int \frac{dy}{(1+y)^2} = \int \frac{dx}{1+x^2}$

$$\frac{-1}{1+y} = \arctan x + C$$

$$\frac{-1}{\arctan x + C} = 1+y$$

$$\frac{-1}{\arctan x + C} - 1 = y$$

$$\frac{-1}{0 + C} - 1 = y(0) = 0$$

$C = -1$

$$y = \frac{-1}{\arctan x - 1} - 1$$

(c) $y = -1$

(5) $X(t)$ = ~~kg~~ of salt in tank at time t.

$X(0) = 100$

$$\frac{dX}{dt} = 0 - 5\,\frac{L}{sec}\,\frac{X\text{ lbs}}{1000\,L} \qquad \frac{dx}{dt} = -\frac{x}{200}$$

In

$X(t) = 100\, e^{-\frac{1}{200}t}$

$10 = 100\, e^{-\frac{1}{200}t}$

$\ln \frac{1}{10} = \frac{-1}{200}t$

$200 \ln 10 = t$

$460.51 \text{ sec} = t$

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