Laplace Transform Facts

(1) $$\mathcal{L}(f(t)) = F(s) = \int_0^\infty e^{-st} f(t)dt$$

(2) $$\mathcal{L}(f'(t)) = s\mathcal{L}(f(t)) - f(0)$$

(3) $$\mathcal{L}^{-1}\left(\frac{F(s)}{s}\right) = \int_0^t \mathcal{L}^{-1}(F(s))|_\tau d\tau$$

(4) If $\mathcal{L}(f(t)) = F(s)$, then $\mathcal{L}(e^{at}f(t)) = F(s-a)$.

(5) $\mathcal{L}(f)\cdot\mathcal{L}(g) = \mathcal{L}(f*g)$, where $(f*g)(t) = \int_0^t f(\tau)g(t-\tau)d\tau$.

(6) $$\mathcal{L}(tf(t)) = -\frac{d}{ds}\mathcal{L}(f)$$

(7) $$\mathcal{L}\left(\frac{f(t)}{t}\right) = \int_s^\infty \mathcal{L}(f)|_\sigma d\sigma.$$

(8) If $u(t) = \begin{cases} 0 & \text{if } t < 0, \text{ and} \\ 1 & \text{if } 0 \le t, \end{cases}$ then $\mathcal{L}(u(t-a)f(t-a)) = e^{-sa}\mathcal{L}(f(t))$.

$f(t)$	$F(s)$
1	$\frac{1}{s} \quad (s > 0)$
t	$\frac{1}{s^2} \quad (s > 0)$
$t^n \ (n \geq 0)$	$\frac{n!}{s^{n+1}} \quad (s > 0)$
$t^a \ (a > -1)$	$\frac{\Gamma(a+1)}{s^{a+1}} \quad (s > 0)$
e^{at}	$\frac{1}{s-a} \quad (s > a)$
$\cos kt$	$\frac{s}{s^2 + k^2} \quad (s > 0)$
$\sin kt$	$\frac{k}{s^2 + k^2} \quad (s > 0)$
$\cosh kt$	$\frac{s}{s^2 - k^2} \quad (s > \lvert k \rvert)$
$\sinh kt$	$\frac{k}{s^2 - k^2} \quad (s > \lvert k \rvert)$
$u(t-a)$	$\frac{e^{-as}}{s} \quad (s > 0)$

$$\Gamma(x+1) = x\,\Gamma(x)$$

$$\Gamma\left(\tfrac{1}{2}\right) = \sqrt{\pi}$$

Table of Laplace Transforms

This table summarizes the general properties of Laplace transforms and the Laplace transforms of particular functions derived in Chapter 7.

Function	Transform	Function	Transform
$f(t)$	$F(s)$	e^{at}	$\frac{1}{s-a}$
$af(t)+bg(t)$	$aF(s)+bG(s)$	$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$
$f'(t)$	$sF(s)-f(0)$	$\cos kt$	$\frac{s}{s^2+k^2}$
$f''(t)$	$s^2F(s)-sf(0)-f'(0)$	$\sin kt$	$\frac{k}{s^2+k^2}$
$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-\cdots-f^{(n-1)}(0)$	$\cosh kt$	$\frac{s}{s^2-k^2}$
$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$	$\sinh kt$	$\frac{k}{s^2-k^2}$
$e^{at}f(t)$	$F(s-a)$	$e^{at}\cos kt$	$\frac{s-a}{(s-a)^2+k^2}$
$u(t-a)f(t-a)$	$e^{-as}F(s)$	$e^{at}\sin kt$	$\frac{k}{(s-a)^2+k^2}$
$\int_0^t f(\tau)g(t-\tau)\,d\tau$	$F(s)G(s)$	$\frac{1}{2k^3}(\sin kt-kt\cos kt)$	$\frac{1}{(s^2+k^2)^2}$
$tf(t)$	$-F'(s)$	$\frac{t}{2k}\sin kt$	$\frac{s}{(s^2+k^2)^2}$
$t^nf(t)$	$(-1)^nF^{(n)}(s)$	$\frac{1}{2k}(\sin kt+kt\cos kt)$	$\frac{s^2}{(s^2+k^2)^2}$
$\frac{f(t)}{t}$	$\int_s^\infty F(\sigma)\,d\sigma$	$u(t-a)$	$\frac{e^{-as}}{s}$
$f(t)$, period p	$\frac{1}{1-e^{-ps}}\int_0^p e^{-st}f(t)\,dt$	$\delta(t-a)$	e^{-as}
1	$\frac{1}{s}$	$(-1)^{[t/a]}$ (square wave)	$\frac{1}{s}\tanh\frac{as}{2}$
t	$\frac{1}{s^2}$	$\left[\!\left[\frac{t}{a}\right]\!\right]$ (staircase)	$\frac{e^{-as}}{s(1-e^{-as})}$
t^n	$\frac{n!}{s^{n+1}}$		
$\frac{1}{\sqrt{\pi t}}$	$\frac{1}{\sqrt{s}}$		
t^a	$\frac{\Gamma(a+1)}{s^{a+1}}$		