Quiz on Feb. 16

Consider the Differential Equation $\frac{dx}{dt} = x^2-5x+4$

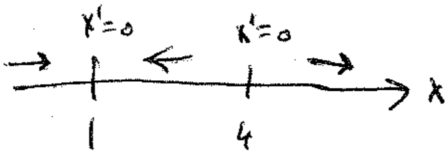
(a) Find all equilibrium solutions of the Differential Equation.

(b) Which of the equilibrium solutions are stable? Which are unstable?

(c) Solve the DE

(d) Graph some of the solutions.

$\frac{dx}{dt} = (x-4)(x-1)$ $\boxed{\text{So } x=4 \text{ and } x=1 \text{ are equilibrium solutions of the DE}}$ a



$0 < \frac{dx}{dt}$ for $4 < x$

$\frac{dx}{dt} < 0$ for $1 < x < 4$

$0 < \frac{dx}{dt}$ for $x < 1$

$\boxed{\text{So } x=4 \text{ is an unstable equilibrium and } x=1 \text{ is a stable equilibrium}}$ b

$\frac{dx}{(x-4)(x-1)} = dt$ $\quad \frac{1}{3}\left[\frac{1}{x-4} - \frac{1}{x-1}\right]dx = dt$ $\quad \ln\left|\frac{x-4}{x-1}\right| = 3t + c$ $\quad \frac{x-4}{x-1} = Ke^{3t}$

$x-4 = (x-1)Ke^{3t}$ $\quad x(1-Ke^{3t}) = 4-Ke^{3t}$ $\quad x(t) = \frac{4-Ke^{3t}}{1-Ke^{3t}}$ $\quad$ of course $K = \frac{x_0-4}{x_0-1}$

So the solution is $x(t) = \dfrac{4 - \frac{x_0-4}{x_0-1}e^{3t}}{1 - \frac{x_0-4}{x_0-1}e^{3t}}$. In other words $x(t) = \dfrac{4 + \frac{x_0-4}{1-x_0}e^{3t}}{1 + \frac{x_0-4}{1-x_0}e^{3t}}$

So $\boxed{x(t) = \dfrac{4(1-x_0)+(x_0-4)e^{3t}}{(1-x_0)+(x_0-4)e^{3t}}}$ c

If $4 < x_0$, then the denominator becomes 0 at some finite time so x goes to ∞ at some finite time.

If $4 = x_0$, then $x(t) = 4$ for all t

If $1 < x_0 < 4$, then $x(t) = \dfrac{4(1-x_0)e^{-3t}+(x_0-4)}{(1-x_0)e^{-3t}+(x_0-4)}$

$1-x_0$ and x_0-4 are both negative,

The denominator is never zero

It makes sense to compute $\lim_{t\to\infty} \dfrac{4(1-x_0)e^{-3t}+(x_0-4)}{(1-x_0)e^{-3t}+(x_0-4)}$

$= \dfrac{0+x_0-4}{0+x_0-4} = 1$

If $1 = x_0$, then $x(t) = 1$ for all t

If $x_0<1$, then $(1-x_0)e^{-3t}+(x_0-4)$ is not zero when $0\le t$

because $(1-x_0)e^{-3t}+x_0-4=0$ occurs only when

$$e^{-3t}=\frac{4-x_0}{1-x_0}$$ but $1<4$

so $1-x_0<4-x_0$

so $1<\frac{4-x_0}{1-x_0}$ (since $0<1-x_0$)

and e^{-3t} is not >1 for $0\le t$

Once again since the denominator is never zero (when $0\le t$) it makes sense to calculate $\lim_{t\to\infty} x(t)=\lim_{t\to\infty}\frac{4(1-x_0)e^{-3t}+(x_0-4)}{(1-x_0)e^{-3t}+x_0-4}=\frac{0+x_0-4}{0+x_0-4}$

$=1$

(d)

Here are some solutions